the local velocity of the mass and that the limits of integration are functions of time

At approximately the same time (although unknown to author), Thorpe[2] established a similart heorem for piecewise continuous masses Thorpe's theorem, although not as general as Bottaccini's, was particularly interesting since he found a practical equation, whereas Bottaccini only proved a principle

Thorpe showed that, for an arbitrary piecewise continuous mass moving in an arbitrary manner under external forces, the forces acting on the portion of the mass within an arbitrarily selected control volume are given by

$$\frac{d}{dt}\int_V \rho \mathbf{U} d\tau = \Sigma \mathbf{F} - \int_S \rho \mathbf{U}(\mathbf{U} - \mathbf{Y})\ d\mathbf{S} \tag{3}$$

in which ρ is the local mass density, V is the arbitrary volume, and $\mathbf{Y}$ is the local velocity of the boundary surface S This relative motion expression can be brought into agreement with Eq (2) by using the definition of momentum given in Eq (1), as was recognized by Thorpe[3] Since in Eq (1) the integral is to be taken over the mass, then the velocity of the bounding surface must be the velocity of the mass on the boundary With this definition, $\mathbf{U} = \mathbf{Y}$, and Eq (3) becomes

$$\frac{d}{dt}\int_V \rho \mathbf{U} d\tau = \Sigma \mathbf{F}$$

For piecewise continuous masses, Eq (1) becomes

$$\mathbf{G} = \int_V \mathbf{U} \frac{dm}{d\tau}\, d\tau = \int_V \rho \mathbf{U} d\tau$$

which shows that Eqs (2) and (3) are identical Equation (3), however, is admirably suited for computations on piecewise continuous masses For highly discontinuous masses and for masses defined on sets of measure zero, the reader is referred to Ref 1

References

[1] Bottaccini, M "An alternate interpretation of Newton's second law," AIAA J 1, 927–928 (1963)

[2] Thorpe, J F, "On the momentum theorem for a continuous system of variable mass," Am J Phys 30, 637–640 (1962)

[3] Thorpe, J F letter to author (July 1963)

Effect of Radiation on Ammonium Perchlorate Propellants

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THIS laboratory has recently completed a series of experiments to determine the effects of radiation on propellants The propellant strands were obtained from the vendors cited and irradiated using a 2-Mev Van de Graaff electron accelerator After exposure to the doses indicated in Table 1, burning rates and tensile measurements were made

It is seen that, in many cases, drastic changes in burning rates and tensile strengths occurred upon radiolysis The mechanisms by which these changes aie brought about are being studied in a continuing program in which the individual components of the propellant iecipe are being irradiated and incorporated into nonirradiated mixes

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Table 1 Effect of radiation on ammonium peichlorate propellants

Propellant	Radiation dose, mrad	Burning rate,[a] in /sec	Tensile strength,[b] psi
Polysulfide,	0	0 0593 ± 0 0006	249 ± 11
Thiokol TP-L 3014	10	0 0593 ± 0 0060	156 ± 9
	50	0 0549 ± 0 0025	51 ± 13
Polysulfide,	0	0 0582 ± 0 0003	136 ± 4
Thiokol TP-L-3014a	20	0 0548 ± 0 0005	138 ± 15
	50	0 0568 ± 0 0006	62 ± 6
Hydrocarbon,	0	0 0422 ± 0 0003	91 ± 4
Thiokol TP-H-3062	20	0 0428 ± 0 0004	168 ± 7
	50	0 0425 ± 0 0004	145 ± 7
Polyurethane,	0	0 0347 ± 0 0002	54 ± 3
Thiokol TP-6-3129	20	0 0355 ± 0 0002	56 ± 3
	50	0 0371 ± 0 0004	40 ± 2
Polyacrylonitrile,	0	0 0660 ± 0 0025	190 ± 8
Heicules HES 6648	10	0 0700 ± 0 0024	72 ± 2
	50	0 0860 ± 0 0027	56 ± 2
Polyethyl acrylate,	0	0 0412 ± 0 0004	111 ± 10
Hercules HES 6420	10	0 0447 ± 0 0005	67 ± 6
	50	0 0486 ± 0 0010	30 ± 4
Cellulose acetate,	0	0 0325 ± 0 0010	541 ± 75
Hercules HES 5808	10	0 0323 ± 0 0006	341 ± 34

[a] Number of determinations = 10–20
[b] Number of determinations = 5

Shell Buckling and Nonconservative Forces

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IN a recent note Niedenfuhr[1] advanced the suggestion that the wide scatter of observed buckling loads of pressurized shells might be attributed to the presence of nonconservative generalized forces The suggestion was based on the statement that a mechanical system that is acted upon by nonconservative generalized forces may buckle dynamically as well as statically This statement, in turn, was supported by the example of a two-degree-of-fieedom system subjected to one conservative and one nonconservative load

It appears to these writers that the statement just quoted, which has limited validity, is not applicable in the sense envisaged by Niedenfuhr, as will be indicated below For a given ratio of the two loads introduced, it is of course possible to calculate a static and a dynamic load, but the physically meaningful one, in general, will be only the lower one If it is the static one, the system will be displaced into a position of static equilibrium corresponding to the actual value of the (supercritical) load If, on the other hand, the stability is lost dynamically, under a load larger than the critical one, the system will vibrate with a definite frequency and with an exponentially increasing amplitude until failure is reached Thus, in the case considered by Niedenfuhr there is no possibility of natural experimental scatter for fixed loading ratios

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For very special loadings it is possible, however, that more than one value of a critical force is physically meaningful This was brought out in a recent investigation of the present writers,[2] in which it was found that a two-degree-of-freedom system may possess multiple stable and unstable ranges of the load The number of physically meaningful critical values of the load has to be odd, because, before a higher critical value is reached, the system, under gradually increasing loads, has to become stable again

Even such type of loading, however, could hardly be held responsible for the scatter of shell buckling loads As pointed out by Bolotin,[3] not a single experiment has ever been carried out in which buckling would have been produced by a nonconservative static force The fact of the matter is that such forces are quite easily introduced into the analytical treatment of a model by means of arrows, but their realizability in a test presents great difficulties Niedenfuhr expects that fluid pressure forces acting on a shell are nonconservative, but this would be true only if it were possible to exert this pressure over a limited area of the shell surface, without applying any other forces, as discussed more fully in Ref 3

Two further aspects of dynamic buckling under nonconservative static forces render its usefulness even more questionable for the purpose of comparing analytical and experimental buckling results The first concerns the peculiar role of damping played in such systems Even vanishing damping, in general, lowers the buckling load[3] and makes it depend in a two-degree-of-freedom system on the ratio of damping of constants for each generalized coordinate Thus, if the loads were nonconservative, damping should have been included in the analysis

The second aspect is the following In the absence of damping, the dynamic buckling load is characterized by two natural frequencies approaching each other as the loading increases and coinciding at the critical value of the loading It is known, however, from the theory of stability of motion that, whenever two frequencies coincide, the usual stability criteria of Routh-Hurwitz might lead to erroneous results, and then a nonlinear analysis has to be carried out Thus, the buckling loads determined from a "small" vibration analysis might be quite inaccurate, and no good correlation with experiments, even if it were possible to carry them out, is to be expected

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[2] Herrmann, G and Bungay R W , "On the stability of elastic systems subjected to nonconservative forces," J Appl Mech (to be published)

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Reply by Author to G Herrmann and R W Bungay

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HAVING studied the writers' arguments in the preceding comment, the author remains unconvinced of their validity The point of the example in the original note is that, even though the parameters of a system have become such as to render it susceptible to dynamic failure, the system may still be statically stable The dynamic modes of deformation may then provide a mechanism for the system to pass from one branch to another of the static equilibrium locus by paths that do not lie on this locus and that may bypass static critical loads As to the scatter, firstly, the fact that a real system may be susceptible to dynamic failure does not mean that it must fail, merely that it will fail if it is subjected to the proper disturbance The load level at which this disturbance is introduced is generally an indeterminate quantity Secondly, the precise load level at which a system becomes susceptible to dynamic failure can in a real system be affected by assembly details, particularly by the amount of dry friction present It is clear, for instance, that dry friction in the hinge in the middle of the compound bar of the original example will profoundly influence the Beck load for the system It is difficult to judge the appropriateness of the writers' Ref 2 since it has not, as of this writing, appeared in print

The author believes that Bolotin's statement (Ref 3 of preceding comment) here is beside the point The unsteady hydrodynamic forces associated with large local deformations of the shell are surely not completely conservative The only question is the effect of the nonconservative components of these forces Their control or elimination in a test admittedly presents great difficulties, but their realization is almost unavoidable

The term "dynamic buckling" here is perhaps an unfortunate one in that it does not illuminate the mechanism of the failure which is precisely the same as that of subsonic wing flutter Indeed, "flutter buckling" would be a much more descriptive term Making use of the analogy thus introduced, one can envision how the introduction of damping might affect the buckling load either downward by increasing the coupling between modes or upward by adding to the effective stiffness of the system

It is of course true that a nonlinear analysis is necessary to determine the buckled configuration of a system All that a linear analysis can do is to determine the critical loads (and even these may even be affected by the choice of coordinates, as is pointed out in Ref 1) In this connection, however, the following theorem due to Lyapunov gives the engineer some faith in the efficacy of linear analysis

Let $F_i(x_1,x_2, \quad)$ be functions of the dynamical variables which are of at least second degree in the x's, and consider the so-called linearizable system given by $\dot{x}_i = a_{ij}x_j + F_i(x_1,x_2, \quad)$, where the a_{ij} are constants Then, according to Lyapunov, if the linearized system $\dot{x}_i = a_{ij}x_j$ is stable (in the sense of having the real parts of each of its characteristic numbers be negative), the original system is stable no matter what the functions F_i may be

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Calculation of Gravitational Force Components

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THE components of the earth gravitational force are computed as the gradient of an assumed geopotential function When the function is simple, perhaps involving only a few of the zonal harmonics, it and its gradient may reasonably be stated directly in terms of rectangular position co-

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